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Constitutive Model for Concrete: An Overview

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ABSTRACT

In the last three decades, the constitutive modelling of concrete evolved considerably. This paper describes various developments in this field based on different approaches such anelasticity, plasticity, continuum damage mechanics, plastic fracturing, endochronic theory, microplane models, etc. In this article the material is assumed to undergo small deformations. Only time independent constitutive models and the issues related to their implementation are discussed.

Key words: Constitutive modelling, Plasticity, Failure criteria, Continuum damage mechanics, Endochronic theory, Microplane model.

INTRODUCTION

Concrete is a heterogeneous, cohesive-frictional material and exhibits complex non-linear inelastic behaviour under multi-axial stress states. The increased use of concrete as primary structural material in building complex structures such as reactor vessels, dams, offshore structures, etc., necessitates the development of sophisticated material models for accurate prediction of the material response to a variety of loading situations. The new developments which are taking place in the area of concrete technology resulted in new generation of concretes, which are better in terms of performance such as high strength concrete (HSC) (Khaloo and Ahmad (Ju, J.W., 1989), ACI state-of art report (ACI Committee 363., 1984), (Candappa *et al*., 2001), reactive powder concrete (RPC), high performance light weight concrete (HPLC) and self compacting concrete, etc.(Kmita., 2000) and Aitcin further stressed the need for new material models.

Concrete structures are often analyzed by means of the finite element method. Analysis of a tructural engineering problem by finite element method is based on solution of a set of equilibrium equations and a kinematically admissible displacement field. These are supplemented by boundary and initial conditions of a particular problem. These statically and kinematically admissible sets are independent of each other, and to link them material constitutive relations are required (Buyukozturk *et al*., 1985). In recent decades, considerable effort has been undertaken to achieve this goal has resulted in partial success. With the present state of development of computer programs related to finite element method, inadequate modelling of engineering materials in general and concrete in particular is often one of the major factors limiting the capability of structural analysis (Chen., 1982), (Bouzaiene *et al*.,1997).Concrete contains a large number of micro-cracks, especially at the interface between aggregates and mortar, even before the application of the external load. Many theories proposed in the literature for the prediction of the concrete behaviour such as empirical models, linear elastic, nonlinear elastic, plasticity based models, models based on endochronic theory of inelasticity, fracturing models and continuum damage mechanics models, micromechanics models, etc., are discussed in the following sections.

Empirical models

The material constitutive law is, in general gained through a series of experiments (Chen, W.F., 1994). The experimental data is then used to propose

functions, which describe the material behaviour, by curve fitting. Obtaining the experimental data is not so easy. Even for the uniaxial case, there is little information available on strain softening portion and the difficulties are much more in case of multiaxial stress situations. One reason for insufficient experimental information after peak is due to difficulties associated with the testing techniques of materials (Popovics, S., 1970). Many testing machines used for standard compression test apply increasing loads rather than deformation which results in uncontrolled sudden failure after peak load. Several investigators have developed techniques to overcome this difficulty but some of them are costly which require stiff testing equipment which is not available in a normal testing lab (Shah et al.,1984). In most laboratories, cylindrical specimens are used for triaxial testing but the type of loading is unfortunately not truly triaxial in nature. The loading may be. Sometimes these are called untrue triaxial test or false triaxial test. Several investigators tried to develop a true triaxial system where all the three principal stresses can be varied independently and also for obtaining homogeneous state of stress in specimens. Bangash reported experimental results for triaxial compression (see Figure 3).
Another reason for the scarcity of test data is scatter of the test data associated with machine precession, testing technique and statistical variation of material properties from sample to sample.

Figures 1,2 shows a typical uniaxial compressive and biaxial stress-strain curves respectively. Some of the uniaxial stress-strain relations proposed by various researchers are given below:Desayi and Krishan (Desayi *et al.*, 1964)

$$\sigma = \frac{E\varepsilon}{1+\left(\frac{\varepsilon}{\varepsilon_p}\right)^2} \quad ...(1)$$

where Ã, µ are stress and strain tensors, E is Young's modulus, p µ is strain at peak stress. Saenz (Sanez.,1964)

$$\sigma = \frac{E_\varepsilon}{1+\left(\frac{\varepsilon}{E_p}-2\right)\left(\frac{\varepsilon}{\varepsilon_p}\right)+\left(\frac{\varepsilon}{\varepsilon_p}\right)^2} \quad ...(2)$$

where Ep is Young's modulus at peak stress.Smith and Young.

$$\sigma = E\varepsilon^{\frac{-\varepsilon}{\varepsilon_p}} \quad ...(3)$$

$$where,\ e = \frac{E}{E_0}, E_0\ \ inital\ \tan gent\ \mathrm{mod}\, ulus.$$

The European Concrete Committee (CEB) for short-term loading gives a parabola and astraight line up to ultimate strain *u* å as

$$\frac{\sigma_c}{\sigma_u} = \frac{k\eta-\eta^2}{1+(k-2)\eta} \quad ...(4)$$

where ó*c* is the cylindrical compressive strength of concrete.
ó *u* = Ultimate stress

$$\eta = \frac{\varepsilon_u}{0.002},\quad k = \frac{[0.0022(1.1E)]}{\sigma_u} \quad ...(5)$$

The value of a*u* is given between 0.003 and 0.0035

A monotonically increasing uniaxial stress and axial strain equation proposed by Sarginand modified by Attard and Setunge

$$Y = \frac{AX + BX^2}{1+CX + DX^2} \quad ...(6)$$

X, Y refers to stress and strain non-dimensional zed with respect to the correspondingvalues at peak stress. Where A, B, C and D are material constants (Shah., 1984).Richard and Abbott (Richard et al., 1975) proposed a three parameter stress-strain relation

$$\sigma = \frac{E_1\varepsilon}{\left(1+\left(\frac{E_1\varepsilon}{\sigma_0}\right)^n\right)^{\frac{1}{n}}} + E_p\varepsilon \quad ...(7)$$

where E_p is plastic modulus,ó*o* is a reference plastic stress, E_{p1} = *E*"*E* and *n* is a shapeparameter of stress-strain curve.Carreira and Chu [16] proposed a stress-strain relation for reinforced concrete in tension

$$\frac{\sigma_t}{\sigma'_t} = \frac{\beta\left(\frac{\varepsilon}{\varepsilon'}\right)}{\beta-1+\left(\frac{\varepsilon}{\varepsilon_t'}\right)^\beta} \quad ...(8)$$

where stress corresponding to the strainå, σ_t point of maximum stress,straincorresponding to maximum stress σ'_t, β is a parameter depends on the shape of thestress-strain diagram.Mander et al.

$$\sigma = \frac{\sigma_p \frac{\varepsilon}{\varepsilon_p} r}{r - 1 + \left(\frac{\varepsilon}{\varepsilon_p}\right)^r} \qquad ...(9)$$

where σ_p and ε_p are peak stress and strain of confined concrete.

$$r = \frac{E_c}{E_c - E_s}$$

$$E_c = 5000\sqrt{\sigma}, \quad E_s = \frac{\sigma_p}{\varepsilon_p} \qquad ...(10)$$

Gerstle proposed a biaxial stress-strain relation by conducting biaxial compressionTests

$$\tau_{oct} = \tau_p (1 + e^{\left(\frac{-2G_0}{\tau_{oct}}\gamma_{oct}\right)} \qquad ...(11)$$

Go = Initial shear modulus.

τ_{oct} = Octahedral shear stress.

γ_{oct} = Octahedral shear strain.

τ_p = Peak octahedral shear stress obtained from the failure envelope.

Equivalent uniaxial stress-strain relations Chen [8] are also available for biaxial andtriaxial stress conditions of concrete. For biaxial compression

$$\sigma = \frac{E_0 \varepsilon_{iu}}{1 + \left[\frac{E_0}{E_s} - 2\right]\frac{\varepsilon_{iu}}{\varepsilon_{ic}} + \left[\frac{\varepsilon_{iu}}{\varepsilon_{ic}}\right]^2} \qquad ...(12)$$

E_o = Initial tangent modulus of elasticity.

$E_s = \frac{\sigma_{ic}}{\varepsilon_{ic}}$ =Secant modulus at the maximum (peak) compressive stress.

ε_{ic} = Equivalent uniaxial strain corresponding to peak compressive principal stress.

ε_{iu} = Equivalent uniaxial strain.

For triaxial tension and compression

$$\sigma = \frac{E_0 \varepsilon_{iu}}{1 + \left[R + \frac{E_0}{E_s} - 2\right]\frac{\varepsilon_{iu}}{\varepsilon_{ic}} - (2R - 1)\left[\frac{\varepsilon_{iu}}{\varepsilon_{ic}}\right] + R(\frac{\varepsilon_{iu}}{\varepsilon_{ic}})^3}$$

where

$$R = \frac{E_0(\frac{\sigma_{ic}}{\sigma_f} - 1)}{E_s(\frac{\varepsilon_{ic}}{\varepsilon_f} - 1)^2} - \frac{\varepsilon_{ic}}{\varepsilon_f}$$

σ_f, ε_f Coordinates of same point on the descending branch of the stress-equivalent straincurve.

Apart from the above many stress-strain relations specific for ascending branch and fordifferent kind of loading are available in the literature (Popovics.S., 1970 and Chen W.F., 1994).

Linear elastic models

Linear elastic models are the simplest constitutive models available in the literature(Chen, W.F., 1994). In linear elastic models concrete is treated as linear elastic until it reaches ultimate strength and subsequently it fails in brittle manner. For concrete under tension, since the failure strength is small, linear elastic model is quite accurate and sufficient to predict the behaviour of concrete till failure. Linear elastic stress-strain relation using index notation can be written as (Ahmad and Shah)

$$\sigma_{ij} = F_j(\varepsilon_k) \quad \sigma_j = C_{ijkl}\varepsilon_k$$

Where F_{ij} is a function and C_{ijkl} represents material stiffness.

But this simple linear elastic constitutive law is often inappropriate as concrete falls under pressure sensitive group of materials whose general response under imposed load is highly nonlinear and inelastic. Also, in case of reversal of loading, these models fail to predict the concrete behaviour.

Nonlinear elastic models

Concrete under multiaxial compressive stress states exhibit significant nonlinearity and linear elastic models fail in these situations. Significant improvements can be made in this situation using nonlinear constitutive models. There are two basic approaches followed for nonlinear modelling namely secant formulation (Total stress-strain) and tangential stress strain (Incremental) formulation. Incremental stress-strain relation using index notation can be written in the following form.

$d\sigma_{ij} = C_{ijkl}{}^{t} d\varepsilon_{k}$ Here C_{ijkl} is the tangent material stiffness.

Plasticity based models

Classical plasticity based models form a big group in literature in the recent past. The mechanism of material non-linearity in concrete consists of both plastic slip and micro cracking. The large variety of models which are available to characterize the stress-strain and failure behaviour of material under multidimensional stress states (Domingo et al.,Chuan-Zhi et al, Tsai, Richard et al.) have certain advantages and disadvantages, which depend, to a large extent on their particular application. Yield criteria, flow rule and hardening rule are the three corner stones of any plasticity model.In plasticity theory the total strain increment tensor is assumed to be the sum of the elastic and plastic strain increment tensors

$$d\sigma_{ij} = d\sigma_j^e + d\sigma_j^p \qquad ...(13)$$

Hooke's law provides the necessary relationship between incremental stress and elastic strain. The plastic part of the strain increment tensor needs a flow rule to define the direction of plastic flow as explained bellow.

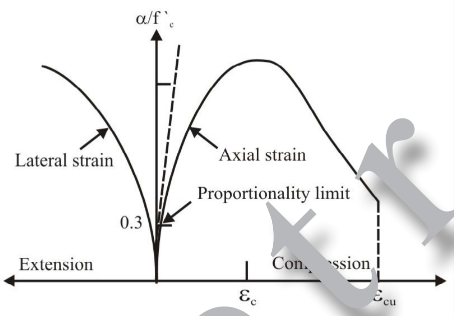

Fig. 1: Uniaxial stress-strain curve

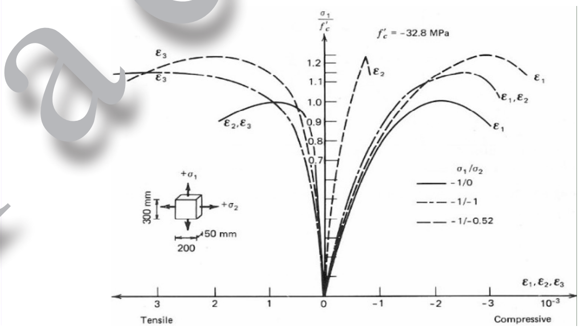

Fig. 2: Biaxial stress-strain curve

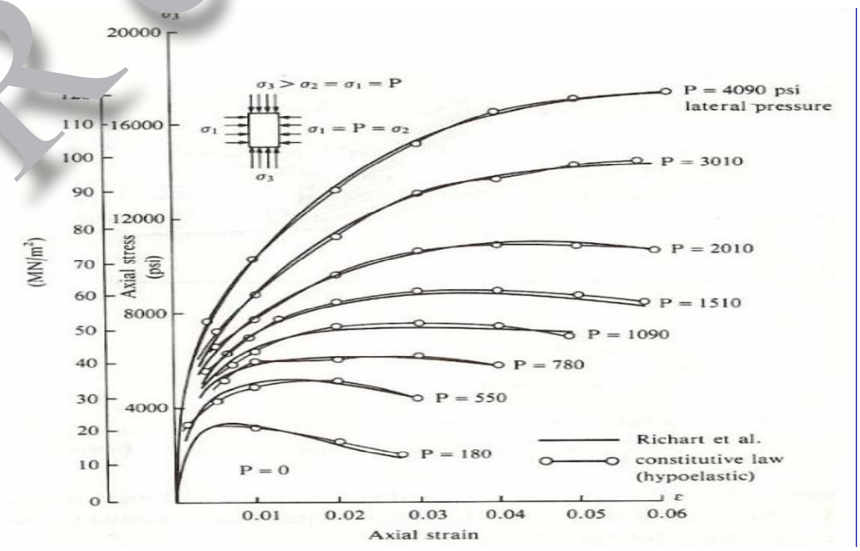

Fig. 3: Triaxial stress-strain curve

Yield criteria

Yield criteria of material should be known from experiments. Bridgman in his experiments pressure showed that hydrostatic pressure has negligible effect on the yield point but this is not the case with all the materials. Concrete is one such material whose behavior is influenced by the effect of hydrostatic pressure. Yield criterion, which are hydrostatic pressure dependent and hydrostatic pressure independent.

Flow rules

A stress increment *d*ó to the current state of stress ó results in elastic as well as plastic strain, if the stress state falls outside the elastic region. To describe the stress-strain relationship for an elastic-plastic deformation, we must define flow rule which define the direction of the plastic strain increment without any information regarding magnitude. Flow rule may or may not be associated with the yield criteria.

$$d\varepsilon_{ij}^{p} = d\lambda \frac{\partial Q}{\partial \sigma_{j}} \qquad ...(14)$$

Where $d\lambda$ is a non-negative scalar; Q is plastic potential function.

Endochronic theory of inelasticity

In the classical plasticity-based models, finding the yield surface pose many problems and an attempt was made to develop a continuous model for inelastic behaviour which did not require the existence of the yield condition. This model is based on the concept of intrinsic(or endochronic) time, defined in terms of strain or stress and used to measure the degree of damage occurred to the internal structure of the material. This model was primarily developed for metals by Valanis. Sandler studied its stability and uniqueness and Rivlin critically evaluated the theory. It has been extended to concrete By Bazant et al, to more reinforced concrete by Reddy and Gopal.

Endochronic model can describe inelastic volume dilatancy, unloading, strain softening, hydrostatic pressure sensitivity and pinching of hysteresis loops under cyclic loading. Even though this model gives superior results, its popularity is restricted by its complexity.

The numerous numerical coefficients required for the development of a constitutive law are estimated by curve fitting of available experimental data. The main obstacle in the development and application of this method is the large number of

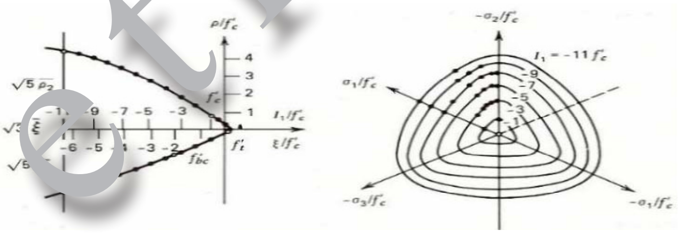

Fig. 4: Willam and Warkne five-parameter model

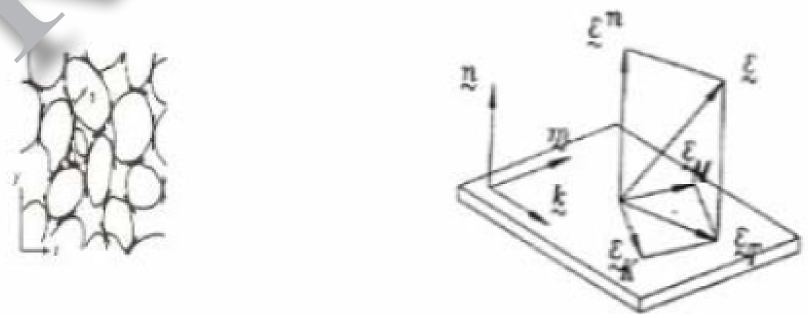

Fig. 5: Microplane and stress-strain components on a microplane

parameters required. As a result, this model has not undergone further development in the last 15-20 years.

The intrinsic time î (on pseudo-time scale) introduced by endochronic theory is

$$\xi = \int_0^{\zeta} \frac{d\zeta}{f(\zeta)}$$

where $f(\zeta) > 0$ and $d\zeta > 0$. The value of $f(\zeta)$ is a history-dependent material function. A typical constitutive equation for linear endochronic theory with pseudo-time measure ζ is as allows (which is similar to a linear viscoelastic model)

$$\sigma_{ij} = \int_0^{\xi} E_{ijkl}(\xi - \zeta')\frac{\partial \varepsilon_k}{\partial \varepsilon'} \qquad ...(15)$$

Fracturing and continuum damage models

These models are based on the concept of propagation and coaleesence of micro cracks, which are present in the concrete even before the application of the load. Damage based models are often used to describe the mechanical behaviour of concrete in tension. In the earlier class of models (Dougill, plastic deformation is defined by usual flow theory of plasticity and the stiffness degradation is modelled by fracturing theory. The second class of models is based on the use of a set of state variables quantifying the internal damage resulting from a certain loading history. The fundamental assumption in these models is that the local damage in the material can be averaged and represented in the form of damage variables, which are related to the tangential stiffness tensor of the material. The models of this category can describe progressive damage of concrete occurring at the microscopic level, through variables defined at the level of the macroscopic stress-strain relationship Krajcinovic and Fonseka. Continuum damage mechanics was introduced by Kachanov in 1958 for creep related problems and has been applied to the progressive failure of materials. In 1980s, it was established that damage mechanics could model accurately the strain-softening response of concrete (Krajcinovic, Lemaitre, Chaboche). Considering the material as a system described by a set of variables and a thermodynamic potential, constitutive law is derived which has to obey the kinematics of damage. Various models of gradually increasing complexity with choice of potential and damage parameter (Scalar, Tensor, etc.) are proposed (Mazars and Cabot, Kratzig and Polling and implemented for concrete. Various damage models such as elastic damage, plastic damage (Ju, Lee *et al*), damage model using bounding surface concept (Voyiadjis),Wu and Komarakul na nakorn presented an endochronic theory of continuum damage mechanics, models for cyclic loading, etc. are available in the literature. Continuum damage mechanics based material models in the literature basically followed two approaches one inspired by plasticity and the other followed the thermodynamic fundamentals and energy balance.In the first approach similar to plasticity, assumes a damage surface, damage loading function and a consistency condition where as in the second approach assumes a free energy potential in the form of Helmholtz or Gibbs subjected to the satisfaction of Clausius Duhem inequality.

CONCLUSIONS

In this article concrete constitutive modelling based on various approaches, their implementation and the aspects related to strain space formulation are discussed. Elasticity based models are simple and material is modelled up to peak. Many attempts for proposing a suitable failure criterion for concrete can be found in literature. These efforts resulted in a realistic failure model such as Willam and Warnke five parameter and subsequently a three parameter model of Menetrey and Willam. These models represent concrete behaviour in a realistic manner.

One advantage of theory of plasticity is the simple and direct calibration of the stress state. The yield surface corresponds to a certain stage of hardening to the strength envelop of concrete, and thus has a strong physical meaning. The theory of plasticity has a very long tradition and hence implementation of the formulation is efficient and thermodynamic validity is assured. One of the disadvantages is the indirect calibration of the deformation behaviour in the form of plastic potential.

Plasticity theory heavily depends on the assumption of existence of a yield surface. This assumption poses a problem while applying plasticity theory to concrete, where a well defined yield surface and experimental data related to yield surface are insufficient. This difficulty gives rise to new theories such as endochronic theory, micro plane theory, etc.

Concrete structures subjected to complex stress states exist widely. Modern analytical tools like finite element method demands a realistic constitutive model. This need has given researchers a chance to explore various approaches such as endochronic theory, continuum damage mechanics, micromechanics, etc. Each of these models has their own strengths and weaknesses as discussed in the above sections.

It is very important to choose a reasonable constitutive model in research and design as it affects the design accuracy to a great extent. More experimental results in complex stress states and more realistic material models are demanded for research and engineering application in the future.

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